Round 1: Elementary Number Theory (NO CALCULATORS)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. What is the **positive** difference between the least common multiple and the greatest common factor of 1440 and 900 ?

2. Write in **base 2**, the result of $(101010)_2 - (3)_{10}$.

3. What prime number is a divisor of every four-digit palindrome?

ANSWERS

- (1 pt.) 1. _____
- (2 pts.) 2. (____)₂
- (3 pts.) 3. _____

Hudson, Notre Dame Academy, and Westborough

Round 2: Algebra 1 (OPEN)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. Subtract and simplify the following expression to a **single** fraction.

$$\frac{5}{2x-2} - \frac{2x-1}{2-2x}$$

2. Working together, Kate and Jean can close the restaurant in $\frac{2}{3}$ of an hour. Working alone, Jean can do the job in 1 hour less time than Kate, when she works alone. How long does it take Jean to do the job alone? Write your answer in terms of hours.

3. If a, b, c, and d are all ≥ 0 and a + c = b, a + d = c, b - d = 2, and b + c - d = 3, determine the sum (a + b + c + d).

ANSWERS

(1 pt.)	1
---------	---

(2 pts.) 2. <u>hr(s)</u>

(3 pts.) 3. _____

Assabet Valley, Doherty, and Southbridge

Round 3: Geometry (OPEN)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. A circle is circumscribed about a square. What is the ratio of the area of the circle to the area of the square?



In the adjacent diagram, triangle ADB is equilateral. The measure of $\ll CAB$ is 60°, $\overline{AC} \perp \overline{CB}$, and $CB = 3\sqrt{3}$. Find the length Of CD.

3. ABCDE is a regular pentagon. What is the degree measure of the acute angle at the intersection of the segments AC and BD?

ANSWERS

(1 pt.)	1.	

- (2 pts.) 2. _____
- (3 pts.) 3. _____

Hudson, Westborough, Worcester Academy

Round 4: Logarithms, Exponents, and Radicals (NO CALCULATORS)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. If
$$\frac{(a^3)^{2n-5}}{(a^3)^{3n-t}} = a^x$$
, determine the value of x in terms of **n** and **t**.

2. $\sqrt[3]{2^{\frac{1}{2}}\sqrt[3]{6^5}}$ can be written in the form $2^p 3^q$. Determine the sum (p+q) and write your answer as a **single** fraction.

3. The solution of $\log_9 x + \log_{27} x = 15$ can be written in the form 3^a . Determine the value of **a**.

ANSWERS

(1 pt.)	1

(2 pts.)	2
----------	---

(3 pts.) 3. _____

West Boylston, Worcester Academy, and St. John's

Round 5: Trigonometry (OPEN)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM UNLESS NOTED BELOW.

1. Express the following expression as a single trigonometric function.

$$\frac{1 - \cos 2x}{\sin 2x}$$

- 2. If $\measuredangle A$ is an acute angle and $\sin 2A = 2x$, determine the sum (sin A + cos A) in terms of x.
- 3. Three circles, centered at A, B, and C, are externally tangent to one another. The circle with center A has radius 3. The circle with center B has radius 5. The measure of $\measuredangle BAC$ is $\frac{p}{3}$ (in radians). What is the measure of $\measuredangle ABC$ in radians to the **nearest hundredth**?

ANSWERS

(1 pt.)	1
· • •	

(2 pts.)	2
----------	---

(3 pts.) 3. ____

Auburn, Tahanto, Bromfield

TEAM ROUND

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND WRITTEN ON THE SEPARATE TEAM ANSWER SHEET. (2 points each)

- 1. The difference of the squares of two prime numbers is 10197. What is the sum of the squares of the two numbers?
- 2. What is the remainder when $x^3 2$ is divided by $x^2 2$?
- 3. What is the length of the altitude drawn to the hypotenuse of a right triangle having legs equal to 5 and 12? Write your answer as a **single** fraction.
- 4. Evaluate $\frac{1}{\log_2 \frac{1}{6}} \frac{1}{\log_3 \frac{1}{6}} \frac{1}{\log_4 \frac{1}{6}}$
- 5. Simplify $\frac{1-\tan^2(\frac{x}{2})}{1+\tan^2(\frac{x}{2})}$ and write your answer as a **single** trigonometric

function.

6. Solve for real x:
$$3^{2x} + 3^x - 20 = 0$$

- 7. Let a, b, and c be nonzero real numbers such that $a + \frac{1}{b} = 5$, $b + \frac{1}{c} = 12$, and $c + \frac{1}{a} = 13$. Determine the value of $abc + \frac{1}{abc}$.
- 8. What is the largest integer whose prime factors add to 14, assuming that if a prime factor is used more than once, then it adds to the sum more than once?
- 9. Let ABCDEFGHIJKL be a regular dodecagon. Determine the value of $\frac{AB}{AF} + \frac{AF}{AB}$

Burncoat, Doherty, Hudson, Leicester, Quaboag,

Round 1: Elementary Number			
1.	(1 pt.)	7020	
2.	(2 pts.)	(100111) ₂	
3.	(3 pts.)	11	
Round 2: Algebra 1-open			
1.	(1 pt.)	$\frac{x+2}{x-1}$	
2.	(2 pts.)	1 hour	
3.	(3 pts.)	4	
Round 3: Geometry (OPEN)			
1.	(1 pt.)	$\frac{p}{2}$ or $p:2$	
2.	(2 pts.)	3√7	
3.	(3 pts.)	72	

WOCOMAL Varsity Meet ANSWERS

Round 4: Logarithms, Exponents, and Radicals

1.	(1 pt.)	x = -3n-12t or $-3(n-4)$	
2.	(2 pts.)	$\frac{3}{10}$	
3.	(3 pts.)	18	
Round 5: Trigonometry (OPEN)			
1.	(1 pt.)	tan x	
2.	(2 pts.)	$\sqrt{1+2x}$	
3.	(3 pts.)	0.67	

TEAM ROUND (2 pts. Each)

- 1. 10205
- 2. 2x 2 or 2(x-1)
- **3.** $\frac{60}{13}$
- 4. 1
- 5. cos x
- 6. $\frac{\ln 4}{\ln 3}$ or $\log_3 4$ or $\frac{\log_a 4}{\log_a 3}$, for any legitimate base a.
- 7. 750
- 8. 162
- 9. 4

WOCOMAL Varsity Meet

TEAM ROUND

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THIS SEPARATE TEAM ANSWER SHEET. (2 points each)

1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	

WOCOMAL Varsity Meet

TEAM ROUND	School:

Team #:_____

Team Members:

1.	
2.	
3.	
4	
5.	

Total Points for Team Round: _____

Worcester County Mathematics League

WOCOMAL Varsity Meet #4

Coaches' Booklet

March 29, 2006