## Round 1: Elementary Number Theory (NO CALCULATORS)

## ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. What is the positive difference between the least common multiple and the greatest common factor of 1440 and 900 ?
2. Write in base 2, the result of $(101010)_{2}-(3)_{10}$.
3. What prime number is a divisor of every four-digit palindrome?

ANSWERS
(1 pt.) $\qquad$
(2 pts.)
2.

(3 pts.)
3. $\qquad$

Hudson, Notre Dame Academy, and Westborough

## Round 2: Algebra 1 (OPEN)

## ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. Subtract and simplify the following expression to a single fraction.

$$
\frac{5}{2 x-2}-\frac{2 x-1}{2-2 x}
$$

2. Working together, Kate and Jean can close the restaurant in $\frac{2}{3}$ of an hour. Working alone, Jean can do the job in 1 hour less time than Kate, when she works alone. How long does it take Jean to do the job alone? Write your answer in terms of hours.
3. If $a, b, c$, and $d$ are all $\geq 0$ and $a+c=b, a+d=c, b-d=2$, and $b+c-d=3$, determine the sum $(a+b+c+d)$.

## ANSWERS

(1 pt.)

1. $\qquad$
(2 pts.)
2. $\quad \underline{h r(s)}$
(3 pts.)
3. $\qquad$

Assabet Valley, Doherty, and Southbridge

## Round 3: Geometry (OPEN)

## ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. A circle is circumscribed about a square. What is the ratio of the area of the circle to the area of the square?
2. 



In the adjacent diagram, triangle ADB is equilateral. The measure of $\Varangle C A B$ is $60^{\circ}$, $\overline{A C} \perp \overline{C B}$, and $C B=3 \sqrt{3}$. Find the length Of $C D$.
3. ABCDE is a regular pentagon. What is the degree measure of the acute angle at the intersection of the segments AC and BD ?

ANSWERS
(1 pt.)

1. $\qquad$
(2 pts.)
2. $\qquad$
(3 pts.)
3. $\qquad$

Hudson, Westborough, Worcester Academy

## Round 4: Logarithms, Exponents, and Radicals (NO CALCULATORS)

## ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. If $\frac{\left(a^{3}\right)^{21-5}}{\left(a^{3}\right)^{3 n-t}}=a^{x}$, determine the value of $x$ in terms of $\mathbf{n}$ and $\mathbf{t}$.
2. $\sqrt[3]{2^{\frac{1}{2}}} \sqrt[3]{6^{\frac{1}{5}}}$ can be written in the form $2^{p} 3^{q}$. Determine the sum $(p+q)$ and write your answer as a single fraction.
3. The solution of $\log _{9} x+\log _{7} x=15$ can be written in the form $3^{\text {a }}$. Determine the value of $\mathbf{a}$.

ANSWERS
(1 pt.) $\qquad$
(2 pts.)
2. $\qquad$
(3 pts.)
3. $\qquad$

West Boylston, Worcester Academy, and St. John's

## Round 5: Trigonometry (OPEN)

## ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM UNLESS NOTED BELOW.

1. Express the following expression as a single trigonometric function.

$$
\frac{1-\cos 2 x}{\sin 2 x}
$$

2. If $\Varangle A$ is an acute angle and $\sin 2 \mathrm{~A}=2 \mathrm{x}$, determine the sum $(\sin \mathrm{A}+\cos \mathrm{A})$ in terms of $x$.
3. Three circles, centered at A, B, and C, are externally tangent to one another. The circle with center A has radius 3. The circle with center B has radius 5. The measure of $\measuredangle B A C$ is $\frac{\pi}{3}$ (in radians). What is the measure of $\measuredangle A B C$ in radians to the nearest hundredth?

## ANSWERS

(1 pt.) $\qquad$
(2 pts.)
2. $\qquad$
(3 pts.)
3. $\qquad$

[^0]
## TEAM ROUND

## ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND WRITTEN ON THE SEPARATE TEAM ANSWER SHEET.

1. The difference of the squares of two prime numbers is 10197 . What is the sum of the squares of the two numbers?
2. What is the remainder when $x^{3}-2$ is divided by $x^{2}-2$ ?
3. What is the length of the altitude drawn to the hypotenuse of a right triangle having legs equal to 5 and 12 ? Write your answer as a single fraction.
4. Evaluate $\frac{1}{\log _{2} \frac{1}{6}}-\frac{1}{\log _{3} \frac{1}{6}}-\frac{1}{\log _{4} \frac{1}{6}}$
5. Simplify $\frac{1-\tan ^{2}\left(\frac{x}{2}\right)}{1+\tan ^{2}\left(\frac{x}{2}\right)}$ and write your answer as a single trigonometric function.
6. Solve for real x: $\quad 3^{2 x}+3^{x}-20=0$
7. Let $\mathrm{a}, \mathrm{b}$, and c be nonzero real numbers such that $a+\frac{1}{b}=5, b+\frac{1}{c}=12$, and $c+\frac{1}{a}=13$. Determine the value of $a b c+\frac{1}{a b c}$.
8. What is the largest integer whose prime factors add to 14 , assuming that if a prime factor is used more than once, then it adds to the sum more than once?
9. Let ABCDEFGHIJKL be a regular dodecagon. Determine the value of

$$
\frac{A B}{A F}+\frac{A F}{A B}
$$

Burncoat, Doherty, Hudson, Leicester, Quaboag,

March 29, 2006
Round 1: Elementary Number Theory

1. (1 pt.) $\quad 7020$
2. (2 pts.) (100111) $)_{2}$
3. (3 pts.) 11

Round 2: Algebra 1-open

1. (1 pt.) $\frac{x+2}{x-1}$
2. (2 pts.) 1 hour
3. (3 pts.) 4

Round 3: Geometry (OPEN)

1. (1 pt.) $\frac{\pi}{2}$ or $\pi: 2$
2. (2 pts.) $\quad 3 \sqrt{7}$
3. (3 pts.) 72

WOCOMAL Varsity Meet ANSWERS
Round 4: Logarithms, Exponents, and Radicals

1. (1 pt.) $x=-3 n-12 t$ or $-3(n-4)$
2. (2 pts.) $\frac{3}{10}$
3. (3 pts.) 18

Round 5: Trigonometry (OPEN)

1. (1 pt.) $\boldsymbol{\operatorname { t a n }} \mathrm{x}$
2. (2 pts.) $\sqrt{1+2 x}$
3. (3 pts.) 0.67

March 29, 2006

## TEAM ROUND ( 2 pts. Each)

1. 10205
2. $2 \mathrm{x}-2$ or $2(\mathrm{x}-1)$
3. $\frac{60}{13}$
4. $\quad 1$
5. $\cos x$
6. $\frac{\ln 4}{\ln 3}$ or $\log _{3} 4$ or $\frac{\log _{a} 4}{\log _{a} 3}$, for any legitimate base a.
7. 750
8. 162
9. 4

March 29, 2006
TEAM ROUND

## ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THIS SEPARATE TEAM ANSWER SHEET. (2 points each)

1. $\qquad$
2. 
3. $\qquad$
4. $\qquad$
5. $\qquad$
6. $\qquad$
7. 
8. 
9. 

March 29, 2006

## TEAM ROUND

WOCOMAL Varsity Meet

School: $\qquad$
Team \#: $\qquad$

Team Members:
1.
2.
3.
4.
5. $\qquad$

Total Points for Team Round: $\qquad$

# Worcester County Mathematics League 

WOCOMAL Varsity Meet \#4

## Coaches’ Booklet

March 29, 2006


[^0]:    Auburn, Tahanto, Bromfield

